Problem set 1

1. a) An important feature in astronomical infrared spectra is the so-called 3.3 micron (μm) emission. Convert to wavenumbers (cm^{-1}) What kinds of chemical bonds do you think this emission is tracing?

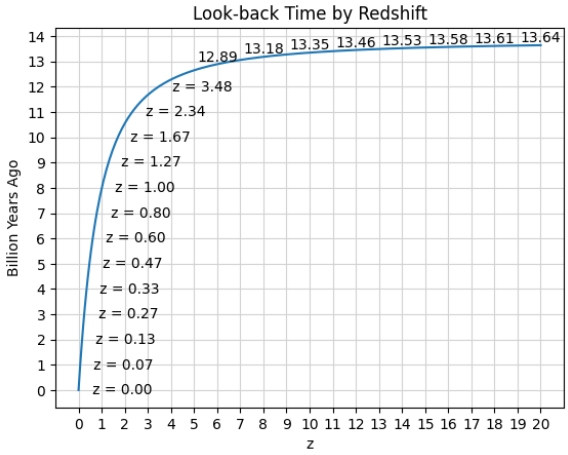
   b) Interstellar dust tends to absorb from 400–5000 cm^{-1}. Convert this range to microns. Do you see a simple rule for converting between microns and wavenumbers?

2. a) An important spectra feature of water vapor observed in interstellar clouds is found at 22.235 GHz. Convert to cm^{-1} and mm.

   b) Can this resonance frequency be classified as sub-mm? What is the frequency range of 0.1–1 mm resonances?

3. H atoms undergo a "spin-flip" at roughly 21.1 cm (called the 21 cm line). What is this frequency in MHz?

4. The Lyman-α line (roughly 121.6 nm) is an important tracer of gas in high-redshift galaxies. Calculate the frequency of the Lyman-α emission from a galaxy formed 9 billion years ago. In what frequency range is it? Suggest a telescope or observing facility that could observe it.



$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$

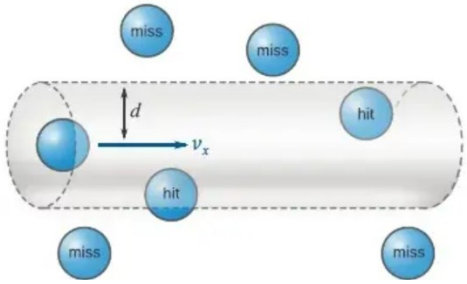
5. Typical air pressure in the Earth's atmosphere is around 1 bar. a) Calculate the air density in cm^{-3}. (cm^{-3} = mL, R = 0.082 Lbar/Kmol, N_A=6.0e+23 mol^{-1}).

b) Ultrahigh vacuum equipment can maintain pressures inside vacuum chambers between 10^{-7} and 10^{-11} mbar or so. How does this pressure compare to pressure conditions in dark cores (ca. 10^5 cm^{-3}).

6. Collisional frequencies between atomic and molecular species in a given volume, per unit time (assuming ideal gas), can be calculated as:

$$Z = N_A N_B \sigma_{AB} \sqrt{\frac{8k_B T}{\pi \mu_{AB}}}$$

where $N_{A,B}$ is the number of A,B particles in the volume, σ_{AB} is the collisional cross section ($=\pi(r_A+r_B)^2$) where r_A is the radius of A and r_B is the radius of B. k_B is the Boltzmann constant, T, is the thermodynamic temperature. μ_{AB} is the reduced mass of particles A and B.



Compare collision frequencies of H atoms (assume V = 1 L and r_H = 1 Å) in Earth's atmoshere, a dense core (10^7 cm^{-3}, T = 10 K), and the diffuse ISM (100 cm^{-3}, T = 100 K).

7. For the following questions about blackbody radiation, go to: https://phet.colorado.edu/sims/html/blackbody-spectrum/latest/blackbody-spectrum_en.html
   a) Judging only by the color of the star above the central graph, estimate the surface temperature of so-called brown dwarfs (named so because of their color).
   b) The interstellar radiation field (ISRF) is thought to mostly be due to radiation from O and B type stars (T > 10000 K). In what part of the spectrum do you expect most of the spectral power density?
   c) Some protoplanetary disks can have temperatures of around 500 K. Estimate the wavelengths of the spectral region covering most of the dust continuum emission.

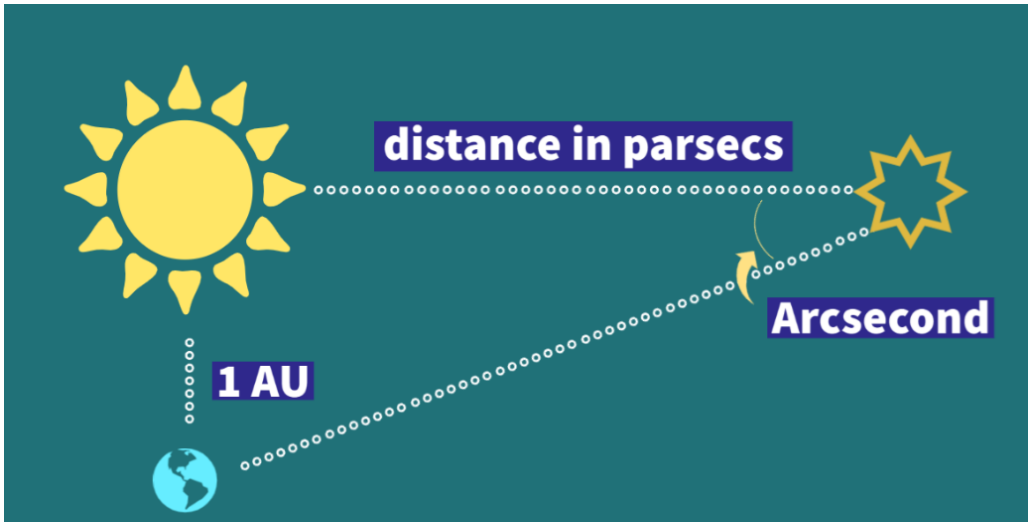
d) What is the λ_{max} of the Sun (in µm and nm)? Why isn't the Sun green?

e) Convert the maximum spectral power density of a 6000 K blackbody (from MW/m^2/µm) to Jy (10^{-26} W/m^2/Hz). Do the same for a 200 K blackbody. In what wavelength range would you expect the Jy to be a suitable unit for line intensity?

8. a) If the speed of light is 2.998e+08 m/s. Calculate the length of a lightyear in km.

b) An Astronomical Unit (AU) is defined as the average distance between the Earth and the Sun (150e+06 km). How many AU are in 1 ly?

c) See accompanying figure for the defintion of a parsec. Calculate how long a parsec is in AU.



9. a) Photodissociation rates are sometimes formulated as:

$$k_{pd} = aexp[-bA_v]$$

where a is the unshielded rate, b is a reaction dependent constant, and A_V is the visual extinction due to dust. The constants a and b are given for two reactions below. Compute the photodissociation rates in a two clouds, one with $N(H_2) = 10^{22}$ cm^{-2} and $N(H_2) = N(H)$, the other with $N(H) = 10^{20}$ cm^{-2}.

Reaction	a	b
CH + $h\nu$ → C + H	2.7exp(–10)	1.3
CH_2 + $h\nu$ → CH + H	5.0exp(–10)	1.7

b) Which reaction is more sensitive to the dust extinction? Why?

c) Electron recombination rates are often given in the form:

$$k_{rec} = \alpha(T/300)^{\beta}$$

Now compare the reaction rates of photoionization and dissociative recombiation of the following reactions

Reaction	a	b
$CH + h\nu \rightarrow CH^+ + e^-$	3.2exp(–10)	3.0
	α	β
$CH^+ + e^- \rightarrow C + H$	1.5exp(–7)	–0.4

Compute the rates of both reactions in the same diffuse cloud (assume $T = 100$ K) and dense core (assume $T = 20$ K) as above. Comment on what you expect from the fate of CH^+ ions.

d) How fast would chemical rates involving CH^+ have to be in order to be relevant in a dense core? How does that compare with the typical Langevin rate?